Petrophysical Properties Prediction Using Self-generating Fuzzy Rules Inference System with Modified Alpha-cut Based Fuzzy Interpolation

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Abstract

Fuzzy systems have been popular in the field of petrophysical properties prediction using well log data. However, the establishment of the fuzzy rule set is a difficult task. A self-generating fuzzy rules extraction technique can be used to set up the fuzzy system. A set of core data with known characteristics is first selected as the training samples. Fuzzy rules are then extracted and undergo a process of rule elimination. This extracted fuzzy rule base will be used to predict properties at other depths within or around the well. One problem always encountered in this technique is missing rules when used in prediction. This problem is caused by fuzzy rule sets that do not cover the whole universe of discourse, leaving gaps in-between the membership functions. This problem has been solved here with the modified α*-cut based fuzzy interpolation technique. The technique interpolates the membership function in the gaps. This work will extend the applicability of fuzzy systems in well log analysis.*

1 Introduction

In petroleum well modelling, boreholes are drilled at different locations around the region. Well logging instruments are lowered into the borehole to collect data at different depths known as well log data [1]. Beside the well log data, samples from various depths are also obtained and undergo extensive laboratory analysis. These laboratory analysis data are known as core data in the well log data analysis process.

Two key issues in reservoir evaluation using well log data are the characterisation of formations, and the prediction of petrophysical properties. Examples of petrophysical properties are porosity, permeability and volume of clay. While a core data set gives an accurate picture of the petrophysical properties at specific depths, it takes a lengthy process and incurs great expense to obtain such data. Hence, only limited core data are available at selected wells and depths.

In well log analysis, the objective is to establish an accurate interpretation model for the prediction of petrophysical characteristics for uncored depths and boreholes around that region [2,3]. Such information is essential to the determination of the economic viability of a particular well or region to be explored. Although empirical formulae relating well log data to the petrophysical properties may be used, the unique geophysical characteristics of each region prevent a single formula from being applicable universally. Instead, statistical techniques and graphical methods are used extensively. To ensure validity of the model, core data from particular wells undergo detailed analysis and serve as referents. Parameters of the model are then manipulated in order to match the overall output to the core data. It is expected that this would result in a better model and increase the overall accuracy.

However, with the availability of increasing number of instruments and log data, it becomes difficult to apply the traditional statistical and graphical methods. To overcome the problem, alternative techniques such as Artificial Neural Networks (ANN) have been applied. Results of these works have been reported in the past few years. Most of the ANN applications are based on the Backpropagation Neural Networks (BPNN) [4,5,6] which make use of core data as training samples. Once the network is trained, it is used as a model to predict subsequent inputs at different depths or boreholes around that region. Although applications of neural networks have been successful, disadvantages such as long training times and the need to select appropriate training parameters have caused inconvenience in practical use. In addition, once the network is trained, the model is seen as a black box and the user has no access to any explicit knowledge that the network has learnt.

This problem may be solved by another technique that could express the function in human understandable rules known as fuzzy systems. A fuzzy set allows for the degree of membership of an item in a set to be any real number between 0 and 1 [7]. This allows human observations, expressions and expertise to be modelled more closely. Once the fuzzy sets have been defined, it is possible to use them in constructing rules for fuzzy expert systems and in performing fuzzy inference. Fuzzy reasoning is expressed as linguistic rules in the form "If x is A, then y is B", where x and y are fuzzy variables, and A and B are fuzzy values. This form of description corresponds well to the rules expressed by humans.

This approach is suitable to this application as the model for each situation may vary greatly and it allows the incorporation of intelligent and human knowledge to deal with each individual case. However, the extraction of fuzzy rules for this application could be very difficult for analysts with little experience. This could be a major drawback for use in well log data analysis.

Recently, an automatic self-generating fuzzy rules inference system [8] had shown successful results in establishing the well log interpretation model. The final interpretation model will comprise of fuzzy rules that the analyst can understand and modify. The user can also add on their experience and knowledge into the fuzzy rules base with ease. However, it suffers from one problem. Depending on the nature of the boreholes, when the extracted rule base is used to predict petrophysical characteristics for uncored depths or boreholes around that region, not all data can find a rule to fire. This is mainly due to the gaps in between the membership functions. In this case, depending on the fuzzy inference program, this may give a very inaccurate prediction. This paper has incorporated the modified α-cut based fuzzy interpolation technique [15, 16] with the self-generating fuzzy rules inference system to solve this problem.

2 Self-generating Fuzzy Rules Inference System

The objective of the self-generating fuzzy rules inference system [8] is to aid the user in setting up a fuzzy rules interpretation model by mapping the available core data to their corresponding memberships. After this has been done, the user can examine the interpretation model from the fuzzy rules. The user can then modify and add-on to the rule base easily. The steps involved in the self-generating fuzzy rules inference system are summarise as follows:

(1) Normalise the data between 0 and 1 by using linear or logarithmic transformations depending on the nature of the well log data.
(2) Define the number of fuzzy regions and fuzzy terms for all data. For ease of extraction, only triangular types of membership functions are used.
(3) The space associated with each fuzzy term over the universe of discourse for each variable is then calculated and divides them evenly.
(4) For each available core data, a fuzzy rule is established by directly mapping the physical value of the variable to the corresponding fuzzy membership function.
(5) Go through Step (4) with all the available core data and generate one rule for each input-output core data pair.
(6) Eliminate repeated fuzzy rules.
(7) The set of remaining fuzzy rules together with the centroid defuzzification algorithm now forms the fuzzy interpretation model.

3 Problem of Self-generating Fuzzy Rules Inference System

To realistically examine the problem of using the self-generating fuzzy rules inference system, we have to work on a real problem. Due to the confidentiality of the data, no borehole details could be published here. Well log data from two typical wells are used to predict the petrophysical property, porosity (PHI). Core data from one well are used to establish a prediction model based on the self-generating fuzzy rules inference system. The model is then used to predict the porosity of the second well. The input logs used in this case study are gamma ray (GR), deep induction resistivity (ILD) and sonic travel time (DT). All the variables are normalised between the values of 0 and 1. The first well has a total of 71 core data and is used to establish the fuzzy rules. The second well has 51 core data and is used as the testing well to test the prediction accuracy of the self-generating fuzzy rules inference system.

A few membership functions (3,5,7,9) are tested, and 9 membership functions appear to give the best prediction results. This is understandable, as more membership functions will cover the approximation function better. Of course, the number of rules will also increase with the increase in the size of the membership function. The total number of rules extracted from the training well is 63. The membership distribution of the input and output are shown in Figure 1.

The prediction accuracy for this case is calculated using the correlation factor as follow:

$$\ell_{x,y} = \frac{\operatorname{cov}(X,Y)}{s_x . s_y} \qquad (1)$$

where $-1 \le \ell_{x.y} \le 1$

and

$$\operatorname{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - m_x)(y_i - m_y)$$

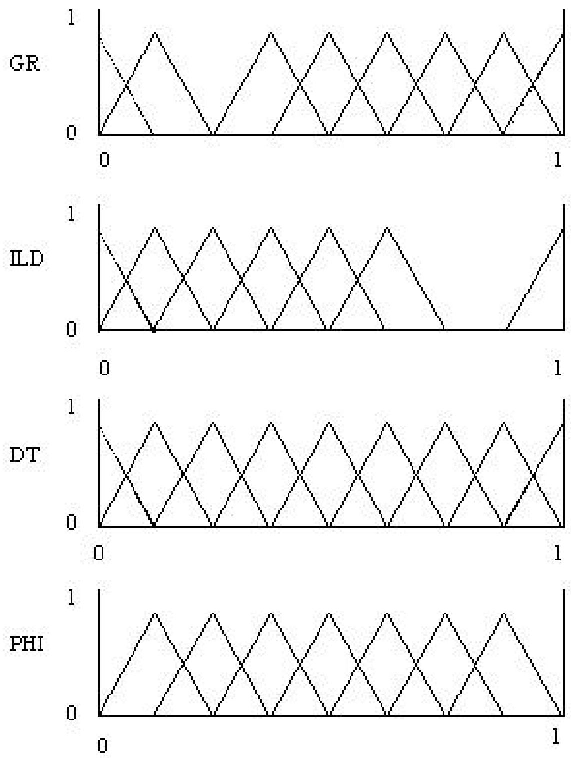

Figure 1: Membership functions for the extracted rules.

The correlation of the predicted training output and the training core data is 0.917; and the predicted testing output and the testing core data is 0.865. The output plot of the predicted testing output (solid line on the plot) as compared to the core data (dots on the plot) is shown in Figure 2.

From the plot, the self-generating fuzzy rules inference system seems to generate promising predictions. However, when performing fuzzy inference on the testing well, two instances of input variables cannot find any rule to fire. They are shown in Figure 3 and highlighted in the circle of Figure 2. From Figure 1, we can also observe that there are gaps in between the membership functions.

In this case study, the number of input sets that cannot find any rule to fire is considered minimal. However, in some cases, this may not always be true. If more than half the input instances in the prediction well cannot find any rule to fire, this interpretation system may be considered useless. This is the major drawback for the self-generating fuzzy rules inference system to be used in petrophysical properties prediction in most practical cases.

However, by observing Figure 1, if some form of interpolation could be used to cover the gaps in between the rule membership functions, then the self-generating fuzzy rules inference system will continue to be useful in the field of well log analysis. The rest of the paper will examine the possibility of using the modified α-cut fuzzy interpolation technique together with this self-generating fuzzy rules inference system.

Warning: no rule is fired for input
[0.271000 0.367000 0.506000]!

Warning: no rule is fired for input
[0.360000 0.322000 0.599000]!

Figure 3: Warning message for input without rule to fire.

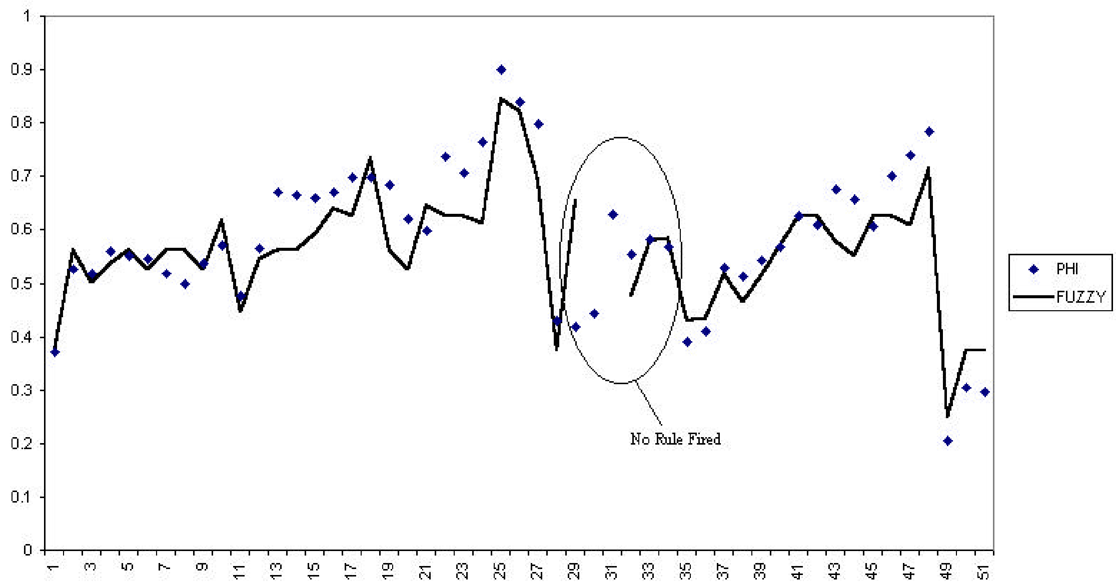

Figure 2: Output plot of testing well.

4 Fuzzy Rule Interpolation

In the case when a rule base contains gaps or is a sparse rule base, classical fuzzy reasoning methods can no longer be used. This is the problem highlighted in the previous section, as an observation finds no rule to fire. Fuzzy rule interpolation techniques provide a tool for specifying an output fuzzy set whenever at least one of the input universes is sparse. Kóczy and Hirota [9] introduced the first interpolation approach known as (linear) KH interpolation.

Two conditions apply for the usage of the linear interpolation. Firstly, there should exist an ordering on the input and output universes. This allows us to introduce a notion of distance between the fuzzy sets. Secondly, the input sets (antecedents, consequents and the observation) should be convex and normal fuzzy (CNF) sets.

The method determines the conclusion by its α-cuts in such a way that the ratio of distances among the conclusion and the consequents should be identical with the ones among observation and the antecedents for all important α-cuts (breakpoint levels).

The KH interpolation possesses several advantageous properties. Firstly, it behaves approximately linearly in between the breakpoint levels. Secondly, its computational complexity is low, as it is sufficient to calculate the conclusion for the breakpoint level set. Moreover, its extension is found to be a universal approximator [10]. However, for some input situation it fails to results in a directly interpretable fuzzy set, because the slopes of the conclusion can collapse. This is shown in Figure 4.

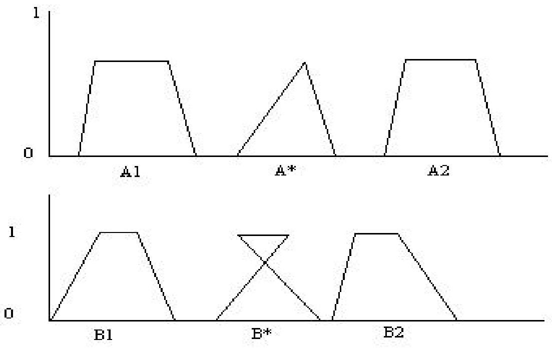

Figure 4: Problem of linear KH fuzzy interpolation.

Several approaches were proposed in the last decade to alleviate this inconvenience [11, 12, 13, 14]. These approaches either determine conditions with respect to the input sets [11, 12] or implement conceptually different method to avoid abnormal conclusion [13, 14]. The new concepts, however, do not preserve the low computational complexity of the original KH method.

Recently, a modification of the original method has been proposed which solves the problem of abnormal conclusion while maintain its advantageous properties [15, 16]. This is known as modified α-cut fuzzy interpolation. This method is selected to incorporate with the self-generating fuzzy rules inference system used in well log analysis. This method works with the vector description of fuzzy sets. The fuzzy set A is represented by a vector $a = [a_{-m}, ..., a_0, ..., a_n]$ where $a_k \, (k \in [-m, n])$ are the characteristic points of A and a_0 is the reference point of A with membership degree one. It means that $a_L = [a_{-m}, ..., a_0]$, and $a_R = [a_0, ..., a_n]$ are the left flank and right flank of A, respectively.

Coordinate transformation is used to avoid the abnormality. The basic idea of the method is that it transforms the space of the consequent sets to another space, where any abnormality can be excluded. The calculation of the conclusion is proceeded in the transformed space, and finally, the resulting set is transformed back to the original space.

The left and right flanks of the conclusion are calculated separately, but their calculations are similar. E.g., for the right flank, the coordinates of the conclusion can be obtained as [15, 16]:

$$b_k^* = {}^{KH}b_k^* + \sum_{i=0}^{k-1} (\lambda_i - \lambda_{i+1})(b_{2i} - b_{1i}) \quad (2)$$

$k \in [0, n]$,

and as for the left flank as:

$$b_k^* = {}^{KH}b_k^* + \sum_{i=0}^{k-1} (\lambda_{-i} - \lambda_{-(i+1)})(b_{2i} - b_{1i}) \qquad (3)$$

$k \in [-m, 0]$,

where b_{1i}, and b_{2i} are the ith coordinate of the consequent B_1, and B_2, respectively; furthermore

$$\lambda_i = \frac{a_i^* - a_{1i}}{a_{2i} - a_{1i}}$$

are the ith ratio factor derived from the appropriate coordinates of the A_1, A_2, and A_*.

$${}^{KH}b_k^* = (1 - \lambda_k) b_{1k} + \lambda_k b_{2k}$$

is the value of the kth coordinate calculated by the α-cut based original KH approach. If only triangular membership functions are used, due to the formula (2) and (3), the left and right flanks of the conclusion are connected at the reference point, $b_0^{'}$.

With the above interpolation characteristics of the modified α-cut based fuzzy interpolation technique, any input variables which fall into the gaps in between the membership can provide some form of interpolated results. This will not only ensure that all input variables can generate reasonable output prediction, it could also ensure that the usage of the self-generating fuzzy rules inference system in the field of well log analysis is practical.

5 Conclusion

In this paper, the practical applicability of the self-generating fuzzy rules inference system in well log analysis has been tested. Based on the real world case presented in this paper, a very undesirable disadvantage of the technique has been discovered. The problem of sparse rule base generated from the core and well logs data causes some undesirable predictions in the uncored depths or borehole around the region. This is mainly due to input instances that could not find any rule to generate reasonable predictions. This is especially important in well log analysis as the nature of boreholes could be very complex.

To solve this problem, the characteristic of the modified α-cut fuzzy interpolation methods has been examined. This method can be used to interpolate the gaps in-between the rules. This ensures that the set of sparse fuzzy rules generated by the self-generating fuzzy rules inference system is more practical to be used in well log data analysis. We intend to examine the results of this technique in another paper. This is useful in the field of petrophysics as it provides another alternative for petrophysical properties prediction that allows more human control.

6 Acknowledgement

This research is supported by the Australian Research Council.

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